Development and Applications of a Semi-Analytical Approximate Thermal Simulator

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ABSTRACT

Oil sand reservoirs play an important role in the economy of Canada due to their significant recoverable reserves. Due to the high viscosity of the oil in these reservoirs, conventional methods cannot be used for production. The steam-assisted gravity drainage (SAGD) method is an efficient way of producing oil from these reservoirs. Predicting oil production and steam injection rates is required for planning and managing a SAGD operation. This can be done by simulating the fluid flow with flow simulation codes, but this is very time consuming. The run time for a 3D heterogeneous model with one well pair can exceed 2 days. Another important task in SAGD operation is the optimization of the trajectory of the wells; the production forecasts for different well positions would require running the flow simulator multiple times, but that is too expensive. Yet another task is to quantify the uncertainty in steam requirements and bitumen production due to multiple realizations of the geological properties. Another task is to rank the multiple realizations from poor performing to good performing. This ranking could be used to help select a subset of realizations for more careful analysis. Finally, forecasting the location of the steam chamber at different time steps is a very important task for considering geomechanical effects. For these reasons, an approximate model that reasonably predicts oil production and steam injection rates with low computational effort would be valuable. In this dissertation, a reliable SAGD approximate simulator for predicting SAGD performance with 3D heterogeneous models of geologic properties is developed. This approximate simulator can handle different types of operating strategies. The approach is an approximate solution using a semi analytical model based on relevant theories including Butler’s SAGD theory. The proxy is much faster than the full simulator and it gives accurate estimated oil production and steam injection rates at different time steps. Theoretical and numerical research has been undertaken to develop the proxy,
implement it in fast code, demonstrate the accuracy of prediction and apply to realistic examples.

**INTRODUCTION**

Based on the Butler's theory, the oil drainage flow at each segment on the interface is equal to the Eq. 1 (Butler, 1987a):

\[ q = \frac{kg\alpha\sin \theta}{mv_sU} = \frac{yg\sin \theta}{mv_s} \]  

(1)

In Eq. 1, \( q \) is rate of oil drainage \( (m^3/s) \), \( g \) is earth's gravity \( (9.81 m/s^2) \), \( k \) is permeability \( (m^2) \), \( \theta \) is angle of segment respect to horizontal direction and \( v_s \) is kinematic viscosity of oil at \( T_s \) \( (m^2/s) \), \( m \) is a dimensionless number equal to the Eq. 2.

\[ m = \left[ v_s \int_{T_r}^{T_s} \left( \frac{1}{v} - \frac{1}{v_r} \right) \frac{dT}{T - T_r} \right]^{-1} \]  

(2)

where \( v_r \) is kinematic viscosity of oil at \( T_r \) and \( v \) is kinematic viscosity of oil at \( T \). Also, \( \gamma \) is the heat penetration depth \( (m) \). Butler obtained a rate of heat accumulation ahead of the interface by writing the differential heat equation and considering conduction and the heat that is left behind the interface. Then, if the temperature gradient varies linearly, the heat penetration rate can be obtained using Eq. 3 (Butler, 1987a):

\[ \frac{dy}{dt} = \frac{2}{\pi} \frac{\alpha}{\gamma - U} \]  

(3)

In this equation \( U \) is velocity of steam chamber advancement \( (m/s) \) and \( \alpha \) is thermal diffusivity \( (m^2/s) \). By combining Eq. 1 and a material balance equation and connecting the steam interface curves to the production wells, oil production rate can be obtained:

\[ q = \sqrt{\frac{1.5\phi \Delta S_o kg \alpha h}{mv_s}} \]  

(4)

Butler called this model as TANDRAIN model (Butler, 1987a, 1991).

For finding the steam interface location, the initial steam interface should be discretized by different segments above the well. By changing time, the steam interface location can be obtained from the material balance equation assuming that the average flow of oil depleting the top section is equal to the flow calculated for each top element. Butler showed that Eq. 5 can be used for finding the interface location (or velocity) before confinement:

\[ \left( \frac{\partial y}{\partial t} \right) = -\left( \frac{\partial Q}{\partial y} \right) \phi \Delta S_o \]  

(5)

where \( \phi \) is porosity and \( \Delta S_o \) is initial oil saturation minus residual oil saturation.

Also, he showed that when the reservoir is not infinite, that is, there is coalescence (contacting steam chambers of adjacent wells) or a reservoir boundary, the steam interface direction should be changed and it moves downward when it reaches the boundary. In this case, the \( x \) location of each segment of the steam interface is the same as before but the \( y \) location can be obtained from Eq. 6.

\[ \left( \frac{\partial y}{\partial t} \right) = \left( \frac{\partial Q}{\partial x} \right) \frac{\phi \Delta S_o}{\Delta S_o} \]  

(6)

Based on Eqs. 5-6, the steam interface location in homogeneous models at different time steps can be plotted. Fig. 1 shows interface curves during the spreading and confinement periods using TANDRAIN theory (Butler, 1987a).

![Figure 1: Interface curves using TANDRAIN theory](image-url)
Butler assumed that the rising steam chamber is cone shaped with the center of the cone at the producer (Butler, 1991). The sides of this chamber are straight lines with an angle of 58 degree from the horizontal. He considered a homogeneous model and computed the amount of hydrocarbon volume in the chamber at differential time $dt$ and set it equal to Eq. 4. Then by integrating the equation, the height of the steam chamber as a function of time can be obtained. Based on Butler’s model, the height of the steam chamber can be computed using Eq. 7 during the rising period.

$$h = 2 \left( \frac{k g \alpha}{m v_s \phi \Delta S_o} \right)^{1/3} t^{2/3}$$

(7)

Butler’s model is efficient for forecasting the steam chamber location of homogeneous models. Due to heterogeneity and existence of shale barriers in the model, the steam chamber location for heterogeneous models is not simply based on the idealized rising, spreading and confinement periods. Multiple rising and spreading periods can be observed in the model and some of spreading periods can start before the rising periods are finished.

Different authors modified the Butler’s original theory to improve the results, but none of them considered heterogeneity effect or different operating strategies (Reis, 1992; Akin, 2005; Edmunds and Peterson, 2007; Miura and Wang, 2012; Gupta and Gittins, 2012; Sharma and Gates, 2011). Vanegas et al. (2008) added different options to the Butler SAGD model for considering heterogeneity of reservoir parameters. They computed the arithmetic average of porosity, permeability, water saturation and diffusivity coefficient were calculated along the steam interface weighed by the distance from the producer to account for model heterogeneity. In this case, they considered different values for porosity, permeability and water saturation at different time steps for computing the oil production rate using Butler’s theory. This model cannot consider the effect of shale barriers in an efficient way. Hampton et al. (2013) considered effect of thermal conductivity and permeability heterogeneity on the SAGD performance. They considered numerical simulation results of the several heterogeneous models and based on those results modified the Butler’s equations for better prediction of SAGD performance by the proxy model. They noticed that variation in permeability more significantly impacted the steam chamber than corresponding thermal conductivity variations. They also noticed that upscaling heterogenic values for input into the proxy analytical model will result in an underestimated flow rate due to the inability to fully account for the impact of shale barriers during the SAGD process.

Dehdari and Deutsch (2013); Dehdari (2014) added many different options to Butler’s model to improve the estimation of SAGD performance from heterogeneous reservoir models with different operating strategies. These are:

1. Develop a new rising model
2. Considering reservoir heterogeneity for forecasting oil production rates
3. Volumetric computation of produced oil behind the steam chamber
4. Consider thief zone effect on the steam injection rate
5. Set injector constraints on the proxy
6. Production trigger for dropping pressure
7. Consider coalescence effect
8. Automatic calibration of proxy results
9. Average relative permeability for flow of oil
10. Consider multiple well-pairs in a DA

After applying all of these modifications, different types of reservoir heterogeneity have been tested with different operating strategies including changes to the steam injection rate, maximum bottom-hole pressure, time varying pressure, ISOR and blow-down trigger. In general, the oil production and steam injection rates and cumulative amounts have been forecast with high accuracy (Dehdari, 2014; Dehdari and Deautsch, 2013).

**About the Simulation Model**

In this paper, 100 3D realizations with the trigger operating strategies have been tested. Three wells should be drilled in the model along the $x$ direction. The grid size in the $y$ direction is increased from 2.5 m to 5 m for increasing the spacing between the wells and testing the effect of changing the grid size on the proxy and simulator results. The grid dimensions are $26 \times 32 \times 83$ and the grid size in the $x$, $y$ and $z$ directions are 25 m, 5 m and 1 m, respectively. Only the reservoir property realizations are changed. Two facies with seven different thermal rock types are considered. Top water and top gas are present in these models. Fig. 2 shows different properties for one slice of one of the generated realizations in the $yz$ plane.
Figure 2: Different properties for one slice of one of the generated realizations in \(yz\) plane—grid sizes in horizontal and vertical directions are 2.5 m and 1 m respectively.

For the operating strategy, the trigger starts working at 600 days and drops pressure 19 times by 100 \(kPa\) each 6 months. The trigger drops the pressure and causes a decreased steam injection rate. The simulator drops the pressure for three months and keeps the pressure constant for the next three months. This process has been simulated by the proxy. Using this trigger, the final cumulative steam injection decreases significantly compared to the base case (no trigger case), but the final cumulative oil production remains close to the base case. All of the models were run through the STARS (Computer modeling group, 2012) and the proxy for finding the oil production and steam injection for 15 years.

In the next section, all 100 realizations are used for finding the optimal trajectory of the producer and injector by maximizing the expected NPV over all realizations after running them with the flow simulator or proxy.

Different constraints have been set for controlling the producer and injector trajectories. These are differences between the elevations of two adjacent completions, maximum distance between minimum and maximum elevations of producer or injector and distance between producer and injector elevations. The distance between the minimum elevation of injector and the maximum elevation of the producer should be greater than zero to prevent steam by-pass. Also it is assumed that any shale barriers between the producer and injector may also prevent oil draining to the producer (Dehdari, 2014).

The objective function of the optimization problem is maximizing cumulative NPV subject to different constraints about location of completions at different slices. In this paper, two methods are tested for well trajectory optimization (Dehdari, 2014). The first method is based on random sampling from a 3D box that has been selected for drilling wells. This method is called the undulate trajectory method (UTM). Then, the differential evolution (DE) optimization algorithm has been used for automatically improving the trajectory location. The UTM method can be very useful and reasonably constrained. The method works good when there are few drilling constraints. Another approach to parameterize a smoother trajectory would be to use a Hermite spline polynomial (Hear and Baker, 2004). In this case, the well trajectory can be defined by four parameters. The starting point of the horizontal portion of the well is called the heel, and the end point of the well is called the toe. Using this method, by knowing the elevations of the toe and heel and also the slope of elevations at these two points, the location of the well trajectory for other slices between the heel and toe can be computed.

In 3D, the well trajectory might be deviated to the left or right for some slices to avoid the BCB elevations or shale barriers. Another spline in the horizontal direction can be considered. This method is called double spline method (DSM). As a result, using such a double spline method, the well trajectory can be optimized in 3D space. This method gives smoother trajectories compared to the first method and drilling the well would be easier. The advantage of the first method is for complex BCB variations and flexible drilling.
The proxy should be called for the objective function instead of the simulator which is thousands of times faster. Although the proxy is very fast, it will be called many times; otherwise, the final value would not be close to the global optimum. The DE optimization method is a population-based method and the rate of convergence to the optimal solution is not fast, but it is suitable for optimization problems with discrete variables. As discussed before, for finding cumulative oil and cumulative steam of a 3D models, Butler’s method assumes each 2D slice separately and after finding cumulative oil and cumulative steam of each slice, all of them should be summed together for finding the performance of the 3D model. In DE optimization algorithm, thousands of 3D well trajectories should be considered and the total NPV using each trajectory must be computed. Although the proxy is very fast with an average run time for a 3D model of about 10 seconds, this can be slow when we want to compute the objective function thousands of times. Also, given uncertainty in the reservoir parameters, the proxy needs to be applied to all realizations. For solving this problem, a 3D array with the size of \((nx_{box}; ny_{box}; nz_{box})\) which \(nx_{box}\), \(ny_{box}\) and \(nz_{box}\) are sizes of 3D box for finding the optimal path of well trajectory can be assumed. Then, the producer should be placed on each grid in that box and NPV should be computed for that producer location. Values of NPV’s should be stored in a 3D array. Then, for finding the total NPV of the 3D model for one trajectory, the NPV value of different completions can be retrieved from this array without any need to run the proxy again. Using this method, the total number of pre-runs would be \(nx_{box} \times ny_{box} \times nz_{box}\) runs of 2D models which can be done very fast. After that there is no need to run the proxy anymore. The following workflow summarizes the optimization of the well trajectory.

1. Define a horizontal box for the optimal location of the trajectory
2. Find 2D gridded BCB elevations and maximum allowable well elevation for all of grids along vertical direction inside of the 2D box of step 1. Using this method, a 3D box can be defined and the NPV for all of grids outside of this box are set to zero. This process should be repeated for all realizations. The horizontal box for all of realizations would be the same, but the vertical ranges would be different, which is due to the difference between BCB elevations
3. Calculate the objective function for all grids inside the 3D box for each realization and then calculate the average of NPV over all of realizations.
4. Sample different trajectories for the producer and injector randomly. If the injector completion in one slice is close to the shale barrier, regardless of producer location, the NPV for that slice would be reset to zero
5. Optimize the producer and injector trajectories simultaneously using the DE algorithm
6. Write the producer and injector trajectories in a file with STARS formats

In this example, 3 well pairs are presented in the model. If elevations of different well pairs are significantly different, the growth of the steam chamber would be uneven. Thus, constraints should be set for the lower and upper elevations of different well pairs and optimal elevations of different well pairs should be computed simultaneously. The NPV of all possible producer locations is computed for all 100 realizations. In this case, the oil price and steam cost are assumed to be \(500 \$US/m^3\) and \(50 \$US/m^3\), respectively. Also, the discount rate is 10% per year. The average NPV over all realizations for all locations could be computed. Then, the DE algorithm can be used for finding the optimal producer and injector trajectories by maximizing the average cumulative NPV using both UTM and DSM methods. The existence of shale barriers at or near the completions will be considered and the best trajectory should be found to maximize the possibility of injecting steam through most of realizations. Fig. 4 shows the optimal producer trajectory of different well pairs in both of UTM and DSM methods on a side and top view. As Figs. 4 shows, the optimal trajectory depends on the parameterization of the trajectory. The UTM method has more flexibility and can find a higher NPV compared to the DSM method. Optimal cumulative NPV for both methods are close to each other. The final cumulative NPV of UTM method is \(1.51E+8 \$US\) and the final cumulative NPV of the DSM method is \(1.50E+8 \$US\). As Figs. 4 shows, it seems that DE optimization algorithm found the best trajectories based on the method and problem constraints by passing the trajectory through hot colors which have higher NPV. Below the minimum elevation of all of the realizations, the NPV for all the grid cells would be zero.

Also, above the maximum elevation of all of realizations, the NPV for all the grid cells would be zero. As Fig. 4 shows, shale barriers are present at different elevations of different slices, and both UTM and DSM methods, placed the optimal location of producer above these barriers to maximize the NPV.
Fig. 4: Optimal producer trajectory of well pair 2 over NPV map for both of methods

Fig. 5 shows the side view and top view of the producer and injector trajectories of different wells for both methods. The injectors are between 4 to 6 meters above the producers and also the minimum elevations of injectors are higher than the maximum elevations of producers to avoid steam by-pass. Also, as the top views show, the location of the injector at each slice can be moved at most one grid left or one grid right of the producer locations for each slice. Also, the constraints on the horizontal and vertical differences between grid cells elevations and horizontal positions have been satisfied in both of these methods. When the trajectories are optimized with more than one realization, the possibility of placing injector completions between shale barriers would be increase – at least on some realizations. For this reason, the optimal injector may moves left or right of the producer location.

Figure 5: Producer and injector trajectories of well pair 2 for both of methods

In the next section, all of these realizations with the optimal well trajectories by the UTM method will be used for ranking and transferring uncertainty of the reservoir realizations.
Uncertainty Transferring and Ranking of the Reservoir Realizations

Considering unlimited steam availability and injecting steam with constant pressure increases the CSOR especially in the last years. When a thief zone exists at the top of the reservoir, the pressure trigger operating strategy can be a good solution to reduce CSOR. In this case, for each well the trigger starts at 600 days and drops pressure $n = 19$ times by $100 \text{kpa}$ during 6 months. As a result, the final cumulative oil production of different realizations would be close to the no trigger case, but the cumulative steam injection and CSOR would decrease significantly. The producer and injector trajectories of the wells in the model are selected from the optimized well trajectories of the last section by the UTM method.

Simulator Results

Fig. 6 shows the results of simulated realizations after running them through STARS and calculating NPV after 15 years of production.

In this case, the oil price and steam cost are assumed to be $500 \text{US}/\text{m}^3$ and $50 \text{US}/\text{m}^3$, respectively. Also, the discount rate is 10% per year.

![Figure 6: Cumulative NPV for different simulated realizations after running them in the reservoir simulator](image)

Ranking Results

Fig. 7 shows a comparison between ranking the reservoir models using the simulator and the other methods after 15 years of production. The correlation coefficient for ranking with OOIP is 0.69 which is very low. The correlation coefficient between the ranked results of the simulator NPV and the proxy NPV is 0.95 which is very good.

![Figure 7: Cumulative NPV for different simulated realizations after running them in the reservoir simulator](image)

The next step is finding the P10, P50 and P90 realizations by the proxy to compare them with the simulator. The central idea of ranking is based on finding these realizations to limit uncertainty of the 100 realizations to 3. Table 1 shows the corresponding simulator ranks for the chosen P10, P50 and P90 realizations based on the proxy after 15 years of simulation.

<table>
<thead>
<tr>
<th>Proxy</th>
<th>Simulator</th>
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<tbody>
<tr>
<td>P10</td>
<td>P4</td>
</tr>
<tr>
<td>P50</td>
<td>P40</td>
</tr>
<tr>
<td>P90</td>
<td>P88</td>
</tr>
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Table 1 shows that the proxy overestimated the simulator rank P10, P50 and P90 realizations and proxy P10, P50 and P90 are less than the simulator P10, P50 and P90 realizations.
Fig. 8 shows a comparison between P10, P50 and P90 realizations of simulator NPVs and proxy NPVs after 15 years of simulation. The proxy results have been shown by the blue curves and the simulator results have been shown by the red curves. Fig. 14 shows that the proxy P10 is lower than the simulator P10 by about 3%. The proxy P50 and proxy P90 are about 2% and 0.1% less than the simulator P50 and P90 respectively. These differences are not significant.

\[ Y = \frac{Z - \mu}{\sigma} \]  

where \( z \) is value in original units, \( \mu \) is the mean and \( \sigma \) is the standard deviation. Fig. 9 shows a comparison between the standardized simulator results and other methods based on their standardized values.

Fig. 9 shows that the ranges of uncertainty around the P10, P50 and P90 realizations for the proxy NPV and proxy oil are very narrow. The ranges for the proxy CSOR are narrow too, but not as narrow as the proxy NPV and proxy oil. The ranges for the OOIP are worse. The mean absolute error (MAE) for identifying P10, P50 and P90 realizations using proxy NPV have been tabulated in Table 2. These values have been obtained by averaging absolute errors between the simulator P10, P50 and P90 with the proxy P10±5, P50±5 and P90±5. As an example, all of values between P45 and P55 have been selected and the mean absolute error with the simulator P50 has been computed. In this case, effect of outliers around P10, P50 and P90 realizations can be studied.

Table 2. Maximum percentage errors for identifying P10, P50 and P90 using proxy NPV

<table>
<thead>
<tr>
<th>Rank</th>
<th>Proxy NPV mean absolute error (MAE)</th>
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<tr>
<td>P10</td>
<td>3.4%</td>
</tr>
<tr>
<td>P50</td>
<td>2.6%</td>
</tr>
<tr>
<td>P90</td>
<td>1.4%</td>
</tr>
</tbody>
</table>

Table 2 shows that even in the worst case, errors are not very large. The largest error is around P10 where there is more scatter compared to the P50 and P90.

After finding the P10, P50 and P90 realizations, P50 realization will be used in the next section for forecasting oil production and steam injection compared to flow simulation results.

**Forecasting SAGD Performance**

The amount of injected steam can be computed based on the heat loss to the steam chamber and produced oil, the heat loss to the reservoir, and the heat loss to the overburden. By
combining these heat losses and using the steam enthalpy, the steam injection rate can be computed. In reality, the steam from one slice could help to produce bitumen in the adjacent slices. In this case, the cumulative oil production and cumulative steam injection would be higher. Due to the long variogram range in the horizontal direction compared to the vertical direction, this effect may not be significant. If the variogram range in the horizontal direction is long and elevations of different completions are not significantly different, this effect can be ignored and steam cross-over between adjacent slices would not be significant.

If the steam chamber in one slice grows quickly to the top of the reservoir in one slice, then steam will enter to the adjacent slices and lose heat to them too. This will increase heat loss for that slice, and as a result, increase the steam injection rate to that slice. This would also decrease the steam injection rate to the adjacent slices. In this case, the steam injection rate of different slices may not be matched, but the predicted cumulative steam injection should be close to the simulator. The proxy can consider the interaction between adjacent slices only when one slice is not able to inject steam or produce the oil of that slice.

The P50 Realization

For testing the proxy efficiency, the P50 model has been tested by the simulator. Fig. 10 shows a comparison between the simulator and proxy for the P50 realization as a 3D example.

As Fig. 10 shows, the proxy final cumulative oil production and cumulative steam injection are close to the simulator. In this case, the final cumulative oil production of the proxy is about 2% less than the simulator result, and the final cumulative steam injection of the proxy is about 4% greater than the simulator result. Also, the oil production and steam injection profiles of the simulator and proxy are close to each other. Except for the first year, the match between the proxy and simulator CSOR is close. The final CSOR of proxy is about 6% greater than the simulator result. The steam injection pressures are matched completely.

Figure 10. Comparison between results of proxy and simulator for the P50 realization
Comparison between the Proposed Work-flow and the Traditional Work-flow

As discussed before, the traditional approach in the SAGD reservoir management is based on the ranking of the reservoir realizations based on the visually optimized well trajectories. For traditional approach, both producer and injector are on the horizontal lines. The producer elevation is 26 m above the base of the reservoir and the injector elevation is 5 m above of the producer. Then, P10, P50 and P90 realizations will be selected for other applications such as transferring uncertainty and well trajectory optimization. Ranking results depends highly on the well trajectory and because of shale barriers around wells, by changing well trajectory ranked results will change significantly. In this paper, all of the realizations have been used for well trajectory optimization. Then, all realizations have been used for ranking and transferring uncertainty using the optimal well trajectories in the first step. Using this approach, the expected NPV of realizations would be higher than the traditional approach. Fig. 11 shows a comparison between the NPV of proposed work-flow and the traditional work-flow. Using the proposed methodology, the expected NPV of the realizations is 16% greater than the expected NPV of the realizations using the traditional work-flow. Also, the P10, P50 and P90 NPV of realizations in the proposed methodology are 16%, 17% and 15%, respectively. Results show a significant increase in the NPV compared to the traditional approach. Although the producer elevation for traditional approach is below the optimal trajectory for the proposed approach for many slices, but the final NPV of the traditional approach is less than the proposed approach.

CONCLUSION

An illustrative case study was performed on a synthetic model with 3 well pairs. This paper proposes a new methodology for SAGD reservoir management. Based on this method, optimal well trajectories have been computed using all of the realizations. After that, all other applications such as, ranking, transferring uncertainty and forecasting SAGD performance have been done using the optimal well trajectories. For well trajectory optimization, it seems that the optimal producer trajectory is close to the averaged BCB for different realizations. In this case, most of the BCBs are below the optimal producer which maximized the expected NPV. The proxy transfers the simulator cumulative oil production and CSOR very quickly and reliably, but the transferred cumulative steam injection of different realizations are less than the simulator which is due to neglecting steam cross over between adjacent slices. As a result, the transferred NPVs are less than the simulator results. Using the proposed methodology, the expected NPV of the proposed reservoir development with the realizations is significantly higher than the expected NPV of the realizations using the traditional work-flow with a few ranked realizations.

Figure 11. Comparison between the NPV of proposed work-flow and the traditional work-flow

REFERENCES


