ABSTRACT

Resistivity borehole images offer a high resolution source of data that is valuable for reservoir characterization. Currently available imaging tools offer high quality measurements with low susceptibility to borehole conditions or artifacts. There is a high correlation between the image measurements and larger scale resistivity logs such as deep induction or laterologs. In formations such as the McMurray formation in Alberta, there is also a strong correlation between resistivity and volume of shale. Borehole images are used to obtain high resolution estimates of fluid saturation, porosity and volume of shale. Coarser scale petrophysical logs provide information to solve the inversion problem along with a shaly-sand conductivity approximation such as the Simandoux equation or Waxman-Smits model. Resulting high resolution properties can be used to construct micromodels of the reservoir for extracting effective permeability at arbitrary scales.

INTRODUCTION

Recording the electrical resistivity of rock has been one of the most widely used measurements in formation exploration and characterization since 1927, when the first resistivity log was recorded by Conrad and Marcel Schlumberger. Related technology has advanced to the point where resistivity measurements can be recorded with an effective sampling volume less a centimeter leading to high resolution logs. Recording many logs for the same borehole provides an annular image of the subsurface that is colored by resistivity rather than light, hence the term formation micro-resistivity image or FMI. There are several variants of formation imaging tools on the market. Weatherford’s high resolution micro-resistivity imaging (HMI) tool is discussed in this paper since it provides a true formation resistivity measurement, as opposed to a differential resistivity measurement that must be corrected when both electrodes are physically connected to the logging tool.

HMI logs provide detailed images for geological interpretation and also for quantifying reservoir properties at a millimeter scale. In siliciclastic formations such as the McMurray formation, there is high contrast between mud and sand so that lithology such as breccia, bioturbation, and sandy to muddy inclined heterolithic stratification can be visually identified (Ranger and Gingras, 2003). Other important features such as unconformities, various bed structures, and fractures are also visible. Properties including dip, volume of shale (Vsh), porosity (ϕ), and permeability (K) can be quantified. A technique called micro-modeling has
been used for deriving \( \varphi \)-K relationships for the McMurray formation (Deutsch, 2010) based on core photographs and FMI. The techniques for estimating \( \varphi \)-K from core photographs were enhanced by Boisvert et al (2012). Enhancements to the workflow involving FMI were done by Zagayevskiy and Deutsch (2012).

Micro-modeling workflows rely on the high contrast between sand and shale through the use of a direct threshold applied to the measurements that result in a binary image. A resistivity threshold is determined to achieve one of two objectives: 1–if a Vsh log is available, the threshold is determined so that the Vsh calculated from the binary image matches the log, or; 2—in the absence of a Vsh log, the threshold is determined to maximize the variance between the sand and shale classes using automatic truncation such as the method of Otsu (1979). An issue with the direct threshold approach and HMI is that resistivity responds to numerous other factors including water saturation (Sw), water resistivity (Rw) and ionic content, shale resistivity (Rsh), \( \varphi \), tortuosity (\( \tau \)) of the current pathway, formation temperature, and borehole wall conditions. None of these factors are sampled at the same resolution as HMI, and some are not sampled at all. Water ions are uncommon and tortuosity cannot be measured with current logging tools. Water saturation and porosity are derived from other logs and have a significantly larger sample volume. Another problem is the Vsh log used to determine the threshold may be subject to error as well, which is a quality control (QC) issue. As with any numerical modeling techniques, the models are only as good as the data used to build them.

In this work, an alternate threshold methodology is used to incorporate other available data as well as several of the factors that affect formation resistivity using a resistivity approximation for reservoirs with shaly-sand depositional systems. The modified Simandoux equation, which is an extension of the Archie equation (Archie, 1942), combines \( \varphi \), Sw, Vsh, \( \tau \), Rw and Rsh and is used to estimate Sw from conventional logs (Bardon and Peid, 1969). Utilizing additional logs along with Vsh should aid in reducing any bias due to QC issues that are present in the logs. Two primary challenges with incorporating the Simandoux equation and Vsh, \( \varphi \) and Sw logs are: 1—the equation is largely underdetermined at the scale of HMI, and; 2—the logs are at a significantly larger scale than HMI. Both are discussed in this paper. An iterative optimization methodology is proposed to achieve scale consistency and satisfy the Simandoux equation at the HMI scale from which a high resolution Vsh image is derived. High resolution \( \varphi \) and Sw images can also be generated with the approach.

**BACKGROUND**

Electrical resistivity of reservoir rocks is a process that is approximated using empirical relationships. One of the first was Archie's law (Archie, 1942) that relates the resistivity of unconsolidated or consolidated sand with the porosity and fluid content. Experimentations led to the empirical formula in Equation (1), where \( R \) is the resistivity of sand, \( \varphi \) is porosity, \( a \) is a tortuosity factor, \( m \) is a cementation exponent, and \( n \) is the saturation exponent.

\[
R = a R_w \varphi^m S_w^{-n} \tag{1}
\]

Archie's law assumes that the second phase when \( S_w < 1 \) is a non conducting phase such as oil or gas. The tortuosity factor was not included in Archie's 1942 work. A relationship between tortuosity and resistivity is given by Madden and Marshall (1959) in Equation (2), where \( \tau \) is the tortuosity that can be observed as the excess length of pore passage that contributes to a resistivity measurement. The same equation was derived by Walsh and Brace (1984) based on effective medium theory. This could be generalized to Equation (1) with \( a = \tau^2 \) and adding in the water saturation for partially saturated media and associated exponents.

\[
R = \tau^2 R_w \varphi^{-1} \tag{2}
\]

Archie's law is widely used in the petroleum industry, most commonly for determining Sw from resistivity logs. In the presence of shale or clay, the relationship overestimates Sw since clay generally leads to a lower resistivity. Several empirical relationships exist that summarize the effects of clay on the resistivity of porous media and in general they take the form of Equation (3), where \( R_{0y} \) is the resistivity of the shaly rock when fully saturated with a conducting phase, \( F = R_{0y}/R_w \) is the intrinsic formation factor and \( C_s \) is the contribution made by the presence of clay (Worthington, 1982).
\[
\frac{1}{R_w^{\prime}} = \frac{1}{F} \left[ \frac{1}{R_w} + C_s \right]
\]

(3)

Three prominent resistivity approximations for shaly-sand are the Waxman-Smits model (Waxman and Smits, 1968), the dual-water model (Clavier, Coates and Dumanoir, 1984), and the modified Simandoux model (Bardon and Pied, 1969). The work by Bussian (1983) and Revil et al (1998) have functional forms similar to Waxman-Smits. They all share similarities with Equation (3). Revil et al (1998) explicitly accounts for the charge carrying capacity of ions in the free electrolyte (water within the pore space) and also the surface conduction due to cation exchange at the water-shale interface. The two regions of conduction are often called the electrical double layer that consists of a diffuse layer in the free pore space and a Stern layer where cation exchange occurs. These models, apart from the modified Simandoux model, involve parameters that are difficult to interpret or require destructive core analysis to obtain.

The modified Simandoux model is defined by Equation (4), where the first term on the right hand side is the conduction due to brine within the pore space and the second term is conduction due to the presence of shale. In the limit as \( V_{sh} \to 0 \) the Archie equation is obtained.

\[
\frac{1}{R} = \frac{\phi^m S_w^n}{a R_w} + \frac{V_{sh} S_w}{R_{sh}}
\]

(4)

Unknown parameters for Equation (4) include \( m, n, a, \) and \( R_{sh}. \) \( R_w \) is often sampled, but not with a high frequency for any given well. It is often assumed constant for each well, or sometimes for an entire field. \( R_{sh} \) is not necessarily sampled. Moreover, it depends on the porosity and tortuosity of the shale, the connate water resistivity, temperature, and pressure (Cremers and Laudelout, 1966; Johnston, 1987; de Lima at al, 2005). Porosity and tortuosity describe the structure of clay that affects resistivity measurements (Fukue et al, 1999). The exponents \( m \) and \( n \) and tortuosity factor \( a \) are also somewhat arbitrary. Values of \( m=2, n=2, \) and \( a=1 \) are typical in sandstones. Values for \( m \) and \( n \) depend on pore structure, cementation, heterogeneity, stress, wettability, and saturation hysteresis. Etris et al (1989) found that \( m \) and \( a \) depended on pore size, pore throat size and connectivity and were likely to vary throughout a reservoir. An inverse relationship between \( \phi \) and \( m \) with \( m \propto \phi^{-1} \) was also noted by Knackstedt et al (2007).

There is no obvious parameter selection for shaly-sands. The parameters are likely to vary across wells and with depth. Various ranges for exponents are summarized in Table 1 from a variety of papers. The tortuosity factor is often ignored and assumed 1. It has been argued that \( a \neq 1 \) is a consequence of fitting Archie theory to non-Archie reservoir rock such as shaly-sand (Herrick, 1988) and is not theoretically justified; however, all functions used in practice that relate resistivity to reservoir properties are either empirical or based on an incomplete understanding of the true underlying physics and are fit experimentally.

**Table 1: Values of constants in Archie, Simandoux, and similar equations**

<table>
<thead>
<tr>
<th>Reference</th>
<th>Model Type</th>
<th>Model</th>
<th>( m )</th>
<th>( n )</th>
<th>( a )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Devarajan et al, 2006</td>
<td>CG</td>
<td>0.89</td>
<td>1.36</td>
<td>1.2 ( \pm 1.54 )</td>
<td></td>
</tr>
<tr>
<td>Greeve et al, 2013</td>
<td>A</td>
<td>1.61</td>
<td>2.14</td>
<td>1.1 ( \pm 5.99 )</td>
<td></td>
</tr>
<tr>
<td>Greenberg and Brace, 1969</td>
<td>CG</td>
<td>2.2</td>
<td>4.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Yue et al, 2005</td>
<td>CG</td>
<td>1.14</td>
<td>1.26</td>
<td>1.03 ( \pm 1.08 )</td>
<td></td>
</tr>
<tr>
<td>Sharma at al, 1991</td>
<td>S</td>
<td>1.22</td>
<td>1.64</td>
<td>1.37 ( \pm 6.01 )</td>
<td></td>
</tr>
<tr>
<td>Knackstedt et al, 2007</td>
<td>A</td>
<td>1.4</td>
<td>2.2</td>
<td>2 ( \pm 5 )</td>
<td></td>
</tr>
<tr>
<td>Fasesan et al, 2007</td>
<td>A</td>
<td>0.251</td>
<td>2.1</td>
<td>1.28 ( \pm 2.5 )</td>
<td>0.76 ( \pm 6.313 )</td>
</tr>
</tbody>
</table>

*\( \phi \) – actual samples; CG – computer generated; S – synthetic (glass beads or mixed soils)

Historically, applications involving resistivity and Archie or Simandoux equations utilize conventional petrophysical logs such as deep induction resistivity. The sampling volume of logging tools used to record resistivity, gamma ray and neutron measurements is on the order of a meter vertically as well as laterally into the reservoir, usually referred to as depth of investigation (Ellis and Singer, 2007). HMI and similar tools record resistivity with significantly smaller sampling volumes, on the order of a cubic centimeter. It is not clear if the same equation relating resistivity, \( \phi, R_w, \) and \( S_w \) applies to both scales, or that it is scale-invariant. Most resistivity approximations are derived at a relatively small scale, for example from core plugs (Bussian, 1983) or from pore-scale numerical simulations (Grattoni and Dawe, 1994; Kozlov et al, 2012). The success of the oil industry indicates that the equations are applicable at the larger scales as well since they are commonly applied to conventional log data.
Micro-modeling utilizes HMI images to reconstruct high-resolution models of the subsurface, where individual grid cells of the numerical models have side lengths of 5 mm to 1 cm. Models are large enough to cover the full diameter of a well bore and full thickness of a formation or interval of interest. In formations like the McMurray, there is a high correlation between resistivity and shale content that has been used to justify the direct truncation of images into binary sand-shale systems. An example that compares HMI with a core photograph is shown in Figure 1. The larger scale of HMI smoothes the true distribution of sand and shale observed in the photograph indicating that truncating the image is inappropriate, that is, Vsh is not binary at the HMI scale. However, obtaining a continuum of Vsh is not straightforward and is the focus of this work. Truncation into binary sand-shale images can lead to biased results, often yielding too high of permeability, particularly in muddy facies. This is due to the smoothness induced by the truncation process, where the boundary between sand and shale is smoother than reality and sands appear devoid of shale features that are visible in smaller scale data sources like core photographs.

Figure 1: HMI image (left) shaded by conductivity (black – low; white – high) and associated core photograph (right).

Once a Vsh image is obtained from an HMI image, the micro-modeling workflow extends the annular image to a high-resolution 3D model. Relationships between Vsh and K and between Vsh and \( \varphi \) are inferred, usually from core samples if available to transform the model into \( K \) and \( \varphi \). Flow-based upscaling is used to recover \( K \) at a scale of interest, typically 5 cm cubes up to 25 cm cubes, since the upper end of bit diameters used in McMurray exploration drilling is 25 cm. Permeability is often anisotropic at a scale larger than HMI samples, so horizontal permeability \( (K_h) \) and vertical permeability \( (K_v) \) are obtained. Symmetric or full permeability tensors can be obtained; however, this is uncommon since most commercial flow simulators rely on diagonal tensors. Arithmetic averaging is used to scale \( \varphi \). Resulting upscaled properties are used to fit relationships, or used to populate larger volumes such as those used in flow simulation (Deutsch, 2010).

**HMI INVERSION**

A continuous field of Vsh can be obtained from an HMI image using inversion, where the underlying reservoir properties are determined so that the HMI image is recovered. The modified Simandoux equation is assumed to be a relevant link between reservoir properties and micro-resistivity. HMI is typically visualized as a conductivity image, so Equation (4) is written using conductivity, \( C \), in Equation (5), where \( C_w \) is the conductivity of water and \( C_{sh} \) is the conductivity of shale.

\[
C = a^{-1} \phi^m S_w^n C_w + V_{sh} S_w C_{sh}
\]

(5)

A typical HMI tool has six pads with 25 buttons per pad that measure resistivity of the formation. Pads are pressed against the borehole wall to minimize drilling mud effects. Resistivity is sampled at 2.5 mm increments as the tool is pulled up the well. Resulting images are 150 pixels in circumference with variable number of pixels along depth. The number of circumferential pixels is denoted \( M \) since the methods could be applied to other tools. Indexing the circumference with \( i \) and depth increments with \( j \), the image is represented as \( C_{ij}, i=1, \ldots, M, j=1, \ldots, N \), where \( N \) is the number of samples along depth. All variables in Equation (5) can be assigned the same indexes that yield an estimate of conductivity, \( C_{ij}^* \). The inversion problem is to solve for all right hand side variables so that \( C_{ij}^* = C_{ij} \).

In previous work, \( C_{ij} \) was truncated to yield binary shale according to Equation (6), where \( t_j \) is a depth dependent threshold that is determined so that the conventional or
coarse scale Vsh log, usually obtained from gamma ray, is reproduced at an appropriate vertical scale. Thresholds for truncation are determined using optimization to minimize squared error, \( (Vsh(j) - Vsh^*(j))^2 \), where \( Vsh(j) \) is the coarse scale log resampled to the resolution of HMI, \( Vsh^*(j) \) is defined by Equation (7), and \( w_k \) are weights that sum to unity and reflect the magnitude of gamma ray emission from the reservoir at a distance associated with \( |k-j| \). The vertical sample volume of \( Vsh(j) \) is equal to \( 2d \), with \( d \) nominally 0.5 m. A Gaussian weighting function is a common choice.

\[
V_{sh}(i, j) = \begin{cases} 
1 & C_{ij} > t_j \\
0 & \text{otherwise}
\end{cases} \quad (6)
\]

\[
V_{sh}^*(j) = \frac{1}{M} \sum_{k=m-d}^{i+d} w_k \sum_{l=w}^{M} V_{sh}(i, k) \quad (7)
\]

The truncation approach also assumes a functional relationship between \( Vsh \) and \( \phi \) at the HMI scale. Two have been used: a linear relationship and a power law (Deutsch, 2010). Using log data, lower and upper bounds on \( \phi \) can be inferred, with porosity in sand, \( \phi_{sa} \), is typically close to 40% and \( \phi_{sh} \) is close to 1% for the McMurray formation. A linear relationship is then defined by Equation (8) and a power law by Equation (9), where \( \omega \) is an exponent that depends on \( Vsh \) and \( \phi_{sh} = 0 \). With truncation, the resulting porosity is also a binary variable taking on values of \( \phi_{sh} \) when \( Vsh = 1 \) or \( \phi_{sa} \) when \( Vsh = 0 \). Assigning fine scale \( Vsh \) and \( \phi \) in such a fashion assumes that all other variability observed in the HMI is due to variations in \( Sw \), which is likely not the case.

\[
\phi = \phi_{sa} (1 - V_{sh}) + \phi_{sh} V_{sh} \quad (8)
\]

\[
\phi = \phi_{sa} (1 - V_{sh})^{\omega(V_{sh})} \quad (9)
\]

Additional coarse scale logs are available to aid in the inversion process, namely \( Sw \), \( \phi \), and true formation resistivity, \( Rt \). True formation conductivity is defined by \( Ct = 1/Rt \). Parameters that were used in the Simandoux equation to determine the \( Sw \) log are also available; however, they are often constants for a well. Logs and parameters provide a coarse scale solution to the Simandoux equation. To further describe the inversion process, a single track of HMI measurements is considered, \( Cij, i \in 1, ..., M, j = 1, ..., N \), that is the equivalent of the fine scale true formation conductivity. To assess the consistency between the HMI and \( Rt \) log, the harmonic average of \( Cij^{-1} \) with a vertical scale equal to the volume sampled by the \( Rt \) log should correlate well (Figure 2). A harmonic average is used in formations that are primarily layered or laminated like the McMurray since the current flows vertically through the formation from the voltage source to the receiver on a resistivity logging tool. This check will also indicate if the micro-resistivity is a true resistivity measurement or a differential resistivity.

![Figure 2: Harmonic average of a single track from HMI compared with a conventional Rt log.](image)

Fine scale \( Vsh \), \( \phi \) and \( Sw \) must be determined to satisfy Equation (5) as well as minimize the squared errors with the coarse scale equivalents: \( ((Vsh(j) - Vsh^*(j))^2, ((\phi(j) - \phi^*(j))^2, and \( ((Sw(j) - Sw^*(j))^2 \), where the fine scale variables are averaged as in Equation (7), but for a single track of HMI. Coarse scale logs and the Simandoux equation provide the necessary known variables and relationship to determine the fine scale reservoir properties; however, there remains 5 fine scale unknowns: \( m \), \( n \), \( a \), \( Cw \), and \( Csh \). Available information for
determining these parameters includes their coarse scale values and ranges observed in the literature. Coarse scale values cannot be used as a solution to the fine scale because the solution to Equation (5) may not exist for the ranges of the reservoir properties, or the solution is physically unrealistic, for example simultaneously requiring a high Vsh and high effective $\varphi$ to obtain the conductivity measured by HMI.

To avoid non-physical solutions, the correlation between variables is accounted for. Vsh and $\varphi$ exhibit a strong negative correlation in depositional environments such as the McMurray formation. Equation 10 is a combined version of Equation (8) and (9) that is assumed to apply at the fine scale, where $\eta_{sh}$ and $\eta_{sa}$ have replaced $\varphi_{sh}$ and $\varphi_{sa}$ since they are defined as constants that can take any value. This allows a more versatile fit that must be truncated to a reasonable porosity range like 0 to 40% ($\varphi_j$) for example. The exponent is dependent on depth rather than Vsh because such a function is fit to data along a well, rather than to specific classes of shale content.

$$\phi = \eta_{sh} + \eta_{sa} (1-V_{sh})^{\eta_j(\cdot)} \quad 0 \leq \phi \leq \phi_1 \quad (10)$$

Using Equation (10), a vector space defined by $0 \leq V_{sh}, S_{w} \leq 1$ can be used to visualize the Simandoux function for various values of the other constants. For example, the typical case where $m=n=2$ and $a=1$ is shown in Figure 3. Rw=1 and Rsh=5 were assumed. The maximum value of 0.235 exists at $V_{sh}=0, S_{w}=1$ for this set of parameters. If these were the coarse scale parameters and Cij=0.5 for example, then the physical space defined by ($V_{sh}, S_{w}$) does not contain the solution to Equation (5); However, if the resistivity of water and shale are adjusted to 0.2 and 2 respectively, then the maximum increases to 0.766. A ($V_{sh}, S_{w}$) pair could be found such that Cij*=Cij=0.5. In fact, there are an infinite number of solutions that exist along a ($V_{sh}, S_{w}$) contour as shown in Figure 3. Since the solution falls in the low Vsh and high Sw region, the shape of this Simandoux function would be appropriate for a region of the well associated with water sands.

Determining the unknown variables and constants to satisfy the Simandoux equation is an iterative optimization process that is similar to Expectation Maximization (Dempster et al, 1977). Constants m, n, a, Cw, and Csh are determined with two objectives: 1—the coarse scale Vsh, $\varphi$, Sw, and Ct logs provide a solution to Equation (5), and; 2—the solution based on HMI (Cij) exists within the pre-defined ($V_{sh}, S_{w}$) space. Objective 1 is a regularization component that provides additional information about the shape of the conductivity surface that was shown in Figure 3. Objective 2 is achieved by ensuring that the inequalities in Equation (11) are met so that a ($V_{sh}, S_{w}$) pair can be found for each HMI measurement that satisfies Equation (5). A lower bound is not necessary since C($S_{w}=0$)=0.
\[
C(V_{sh} = 0, S_w = 1) \geq C_{ij}^*, \forall i, j
\]

\[
C(V_{sh} = 1, S_w = 1) \geq C_{ij}^*, \forall i, j
\]

Constants are constrained somewhat arbitrarily according to: 0.1<\text{m,n}<5, 0.1<\text{A}=\text{Rw}<10, \text{ and } 0.1<\text{Rsh}<50. Ranges for \text{m} and \text{n} are based on what was found in the literature. Lower bounds were reduced since the sands in the McMurray can be quite loosely packed with no cementation. The tortuosity and water resistivity are combined into a single parameter, \text{A}. A value of \text{a}=6 and \text{Rw}=1.67 ohmm results in \text{A}=10 for example, so the range is considered reasonable. Obtaining information on the range for \text{Rsh} was difficult. Shales in the McMurray are often studied for other purposes, such as their physical impact on recovery processes. Johnston (1987) studied physical properties of a North Sea Malm shale that exhibited resistivity up to 45 ohmm; however, such shales cannot be used as a basis due to differences in deposition and mineralogy. It does not seem unreasonable that shale with very low effective porosity, or a highly tortuous conductive pathway could exhibit a high resistivity such as 50 ohmm, especially if the connate water also has low salinity. Patchett (1975) also explored the resistivity of clay gels and various samples with some indication of high shale resistivity; however, the occurrence was infrequent. Kaolinite gels tended to exhibit higher resistivity (20 – 30 ohmm) at low porosity than other types of clay due to its lower cation exchange capacity. More research is required in this area.

A randomized search strategy is used to solve for the constants and satisfy the two objectives. States are selected within the specified ranges using uniform random numbers with a variance and mean that is adjusted iteratively towards the state that minimizes the error with the objectives. Having a known set of constants, the fine scale reservoir properties are solved for, also with two objectives that are numbered 3 and 4 to avoid confusion with the previous two: 3–to ensure that the Simandoux equation is satisfied, and; 4–so that when the fine scale properties are upscaled, the error with the known coarse scale logs is minimized. To satisfy objective 3, a double-threshold approach is used to determine \text{Vsh} from the fine scale conductivity followed by a line search strategy to determine the \text{Sw} that falls on the solution contour previously explained.

As with the threshold approach to determine a binary \text{Vsh} field in previous work, the latest approach also determines \text{Vsh} first followed by \text{Sw}. This is due to the shape of the Simandoux function: for a known \text{Vsh}, the function is solved for a single unknown \text{Sw}, whereas for a known \text{Sw}, there may be two \text{Vsh} solutions. The double-threshold approach aims to determine an upper and a lower conductivity threshold, \text{C}_0 and \text{C}_1, respectively, to define \text{Vsh} as a piece-wise linear approximation in Equation (12).

\[
V_{sh}(C, C_0, C_1) = \begin{cases} 
0 & C \leq C_0 \\
C - C_0 & C_0 < C < C_1 \\
1 & C_1 < C
\end{cases}
\]

Two thresholds are used rather than one because at the scale of HMI, \text{Vsh} is a continuum rather than a binary variable, that is, it has a variance less than \text{Vsh}(1-\text{Vsh}) within a given volume. By maximizing \text{C}_1-\text{C}_0, the solution also minimizes entropy (maximizes uncertainty), unlike the binary case that maximizes entropy. This is reasonable since it is not obvious the conductivity that indicates shale or sand, and also due to the plethora of other influential factors such as water conductivity, pressure, grain size distribution, etc. Gradient descent or similar algorithms can be used to solve for the two thresholds with the objective function defined by Equation 13, where an error for \text{r} is not included since it is defined by \text{Vsh} via Equation (10).

\[
\varepsilon = \sum_j (V_{sh,j} - V_{sh,j}^*)^2 + (S_{w,j} - S_{w,j}^*)^2
\]

By minimizing the error of coarse scale \text{Vsh} and \text{Sw} combined, the thresholds are influenced by the parameterization of the Simandoux equation. If \text{m,n,A}, and \text{Rsh} do not fit the data well, then the error for both logs will be high. This provides information for updating the Simandoux parameters, where the error is used to adjust the two objectives previously discussed. In objective 1, the coarse scale logs are essentially function control points and objective 2 is a lower bound on maximum conductivity. The primary control on the shape is through \text{Sw}. In cases where \text{Sw}*>\text{Sw}, the conductivity lower bound is increased so that the estimated fine scale \text{Sw} yields too high a conductivity. Future iterations will lead to lower \text{Sw}*. At the same time, control
points are adjusted to alter the shape of the contours. For $Sw^*>Sw$, the conductivity at control points is increased, which reduces the convexity of the contours going from convex upwards as shown in Figure 3, towards linear or convex downwards. In the opposite case where $Sw^*<Sw$, the lower bound must be reduced; however, it cannot go below $C_{ij}$. Conductivity at the control points is also reduced.

The inversion process is carried out iteratively with the following procedure: 1–constants for the Simandoux equation are determined form coarse scale logs; 2–thresholds are determined to obtain fine scale reservoir properties; 3–error between coarse scale logs and upscaled fine scale logs is calculated and used to update the objectives for step 1. This process is repeated until a good match between upscaled fine and coarse scale logs is obtained, or the changes to the objectives for determining the Simandoux constants are insignificant. The process is computationally demanding since it is applied to all HMI tracks of which there are $M=150$.

**EXAMPLE**

A 3m segment of an HMI image is used to demonstrate the approach. The segment is a combination of shale beds and breccia with intervals of oil saturated sand (Figure 4). It is imperative to clean such images prior to further processing to eliminate artifacts, dead buttons, and voltage variations (Lofts and Bourke, 1999). In Figure 4, the image was cleaned using a median filter followed by removal of signals that appear as lines using a low pass filter and finally a U-shaped filter was used to remove saw teeth artifacts that are obvious along high contrast boundaries (Nixon and Aguado, 2008).

![Figure 4: 3m segment of an HMI log before cleaning (top) and after cleaning (bottom). Depth increases from left to right and conductivity increases from dark (oil saturated sands) to light (water saturated clays) in the top image. The bottom image has been inverted and converted to grayscale.](image)

Figure 5 shows the results after the first iteration. There is a good match with the coarse scale $V_{sh}$ but error with the $Sw$ log indicating the constants in the Simandoux equation must be updated. After seven iterations of updating the constants, the sum squared error calculated over 1300 pixels between the coarse scale $V_{sh}$ and $Sw$ logs was less than 0.5, see Figure 6. Coarse and fine scale $\varphi$ logs also match indicating the functional relationship between $\varphi$ and $V_{sh}$ is performing well, although this may be application specific. Resulting $V_{sh}$ and $\varphi$ images identify the sand and shale zones as well as some of the fainter features. The $Sw$ image shows horizontal bands, which is most likely due to using constants for Equation (5) that vary with depth, but are fixed around the full image. Further work is necessary to refine the $Sw$ solution and ensure it is physically plausible. The visible correlation between $V_{sh}$ and the lower bound on $Sw$ seems reasonable, since increased shale content lends itself to higher irreducible water due to the hydrophilic properties of clay.

**CONCLUSION**

Relationships that explain the electrical conductivity of shaly-sands provide a means to extract reservoir properties from coarse scale and high resolution resistivity data sources. Coarse scale logs in shaly-sand formations such as the McMurray are processed using relationships such as the Simandoux equation, which can also be used to process HMI. A deterministic inversion process was derived that converges on a solution to the constants including $m$, $n$, $\alpha R_w$, and $R_{sh}$ and reservoir properties including $V_{sh}$, $Sw$, and $\varphi$ so that the Simandoux equation yields conductivities that are equal to the HMI. The method is deterministic since the HMI is directly converted to $V_{sh}$ using a double-threshold approach. Resulting $V_{sh}$ and $\varphi$ images could be used in the micro-modeling process to obtain estimates of permeability. An interesting avenue for further research in this area is the use of stochastic inversion, since the relationship between HMI and $V_{sh}$ is likely not a 1 to 1 function. Assumptions about the variogram of fine scale properties and change of support laws could be utilized to generate realizations of properties and constants that would be updated in such a manner to converge to a match between a resistivity equation and the observed HMI.
Figure 5: Initial iteration of the HMI inversion workflow. Bold black lines are target coarse scale logs. Red curves are upscaled fine scale images. Light lines are 6 curves, one from each pad converted to the associated properties. C is conductivity showing 6 curves, one from each HMI pad, where predicted conductivity in red are nearly invisible since the properties and constants satisfy the Simandoux equation. Porosity is shaded from 0 (blue) to 0.4 (red) and Vsh and Sw from 0 (blue) to 1 (red) using a typical jet color palette.
Figure 6: Seventh and final iteration of the HMI inversion workflow. Bold black lines are target coarse scale logs. Red curves are upscaled fine scale images. Light lines are 6 curves, one from each pad converted to the associated properties. C is conductivity showing 6 curves, one from each HMI pad, where predicted conductivity in red are nearly invisible since the properties and constants satisfy the Simandoux equation. Porosity is shaded from 0 (blue) to 0.4 (red) and Vsh and Sw from 0 (blue) to 1 (red) using a typical jet color palette.
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