Uncertainty, sensitivity and rejection in predictive reservoir forecasting

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This paper has been selected for presentation for the 2014 Gussow Geosciences Conference. The authors of this material have been cleared by all interested companies/employers/clients to authorize the Canadian Society of Petroleum Geologists (CSPG), to make this material available to the attendees of Gussow 2014 and online.

ABSTRACT

In reservoir management, uncertainty modeling and quantification is a key component. Common practice in industry is to use the available data as the starting point (basis) of the reservoir modeling process. Data-driven model construction often leads to an anchoring effect on the base case scenario, leading to unrealistically low uncertainty and overconfidence in the data. Here, we propose to revisit the approach to reservoir modeling and move away from matching as a purpose of the modeling process to forecasting as a purpose. The workflow is based on distance-based modeling and metric space.

INTRODUCTION

Realistically assessing reservoir uncertainty is one of the important challenges in reservoir engineering. Current practice consists of building a set of models matching the data, then employing these models to assess uncertainty on some prediction variable. In an effort to create models that match the data, modelers tend to make assumptions and simplifications that may lead to unrealistic models of uncertainty. For example, although geosciences have a significant qualitative component which describes the reservoir in terms of interpretations derived from data, uncertainty in interpretation (e.g. structural uncertainty and geophysical interpretations) is often ignored. In addition, models are often over-simplified (Gaussian-like models), because they are easier conceptually and may lead to a more rapid match of the data. For history-matching, techniques such as gradient-based optimization or ENKF are much more popular than sampling techniques (Tarantola, 1987). However, these methods aim for efficient reservoir model construction but may not reflect realistic uncertainty. On the other hand, sampling methods aim for realistic uncertainty quantification but are less efficient and thus more time consuming. All these factors may result in unrealistic models of uncertainty, where modelers end up “chasing the data” (Fig. 1, left) continuously and are required to rebuild models to reflect newly acquired data. However, models matching all data do not guarantee accuracy in prediction/purpose.

In our research, we assert that the modeling exercise should start with the widest possible prior model and then
narrow the uncertainty ranges based on the data and the prediction variable (Fig. 1, right), consistent with the Bayesian approach. In addition, because any modeling of uncertainty is irrelevant without a prediction or decision goal, models of uncertainty should be generated with the purpose of the model in mind. To that end, we use the distance-based modeling approach (Scheidt and Caers, 2009) which allows to incorporate the purpose of the modeling in the workflow. Distance-based modeling can also account for the various types of uncertain parameters (from interpretations to numerical values) in a consistent manner.

In this research, we propose to start the modeling exercise with a set prior models generated from a wide range of uncertain parameter values, with emphasis on providing geological uncertainty through multiple interpretations/scenarios. Instead of focusing from the outset on the data to build models, the data is first used to improve understanding of uncertain parameters or reject scenarios that were initially thought possible but that are subsequently shown to be inconsistent with the data. An understanding of what impacts flow processes and decision variables is needed, thus we propose to perform a sensitivity analysis and analyze the effect of each parameter or combination of parameters on the response of interest. Sensitivity analysis requires a link between flow processes and decision variables, where uncertain parameters could be continuous or discrete numerical values, or non-numerical values such as scenarios/interpretations. A distance-based modelling approach is used to link the model parameters and responses. Subsequently, uncertainty quantification must be able to update the probability of multiple scenarios/interpretations that were shown to be influential, given the available data. Again, a distance-based modeling approach is used to represent the set of prior models and the data in a low-dimensional space which facilitates the updating of the probabilities of the uncertain parameters. The final step consists of matching the data, whether seismic and/or production data while accounting for the updated uncertainty in the various modeling variables. In this paper, we use a sampling approach to generate models, since we wish to ensure a realistic quantification of uncertainty.

Each step of the proposed approach (illustrated in Figure 2) is described in the following sections, using examples derived on a real field in West Africa (WCA).

**PROPOSED STRATEGY FOR UNCERTAINTY MODELING**

Uncertainty modeling is usually performed in a Bayesian framework. This implies that the prior model should ideally be constructed before examining any data, otherwise the approach is not strictly Bayesian. Although we recognize that this is not feasible, the prior model must include as much uncertainty as possible, including uncertainties in interpretations. Matching the data should be de-emphasized, as it may lead to reduced uncertainty. This suggests that “rejection” (Popper, 1959 and Tarantola, 2006) should be used instead of matching: prior models that are not compatible with the data should be rejected based on the data. In addition, any modeling of uncertainty is irrelevant without a prediction or decision goal. As a consequence, models of uncertainty should be generated with the purpose (prediction) of the model in mind. Moreover, we must recognize that obtaining the decision variable may require the evaluation of a model which is CPU intensive. The strategy that we elaborate must take this into account.

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**Figure 1.** (left) Current practice in modeling uncertainty: chasing the data, (right) a desired focus on uncertainty
West Coast Africa field case

The West Coast African (WCA) reservoir is a deep water turbidite offshore reservoir located in a sloped valley. The reservoir has been extensively described by Scheidt and Caers (2009), Park et al. (2013) and Fenwick et al. (2014) and here a short summary is provided. One major source of uncertainty in this model is the facies description, which cannot be easily inferred from the seismic data. However, four depositional facies are interpreted from the well logs: shale (Facies 1), poor-quality sand #1 (Facies 2), poor-quality sand #2 (Facies 3) and good-quality channel sand (Facies 4). The depositional uncertainty for the facies is expressed through different training images (TI), shown in Figure 3.

DISTANCE-BASED GENERALIZED SENSITIVITY ANALYSIS (dGSA)

Sensitivity analysis should be an integral part of any uncertainty modeling studies: uncertainty is unimportant if it does not affect a decision variable. The need for an understanding and discovery of what impacts flow processes and decision variables is a critical step in the modeling process, and helps to reduce computational demand. Because of the interpretative nature of reservoir model building, an approach to sensitivity analysis that can handle combinations of discrete and continuous parameters is required, as well as a method that can cross disciplines (geology, geophysics, engineering). This is especially important because the most influential variables are often discrete, representing different interpretations of the reservoir data. In addition, the approach for sensitivity analysis should be able to account for spatial uncertainty and time-varying responses, as well as calculate multi-way parameter interactions. An additional challenge in reservoir modeling is that flow simulation can be very CPU intensive.
To address these requirements and challenges, we propose to use the distance-based generalized sensitivity analysis (dGSA) presented in Fenwick et al. 2014. The principle of dGSA is the following: the procedure classifies the response/decision variable into a limited set of discrete classes. The parameter distributions of each class are then compared, and the following principle is applied. If the parameter frequency distribution is the same in each class, then the model response is insensitive to the parameter, while differences in the frequency distribution indicate that the model response is sensitive to the parameter.

The dGSA is applied to the WCA test case. In this study, we have uncertainty in the depositional scenario, which is represented as 6 equiprobable TIs (shown in Figure 3), and 4 continuous parameters in the flow simulation:

- Residual oil saturation: SOWCR \((U[0.15, 0.35])\)
- Maximum water relative permeability value: \(k_{w\text{Max}} (U[0.3, 0.6])\)
- Water Corey exponent: \(w_{\text{Exp}} (U[2, 4])\)
- \(K_v/K_h\) ratio: \(K_v/K_h (U[0.1, 1])\)

Each parameter is given a cdf, specified above, where \(U[X, Y]\) indicates a uniform distribution between numerical values \(X\) and \(Y\), inclusive.

Distance-based classification is used to separate the response variable into different classes (Scheidt and Caers, 2009). The principle is that the distance is tailored to model response/forecast and thus the classification represents differences in model responses/forecasts. For example, a simple Euclidean distance can be used for time-series variables. Based on the pair-wise distance between a set of model responses, a k-medoid clustering approach is applied to create \(N_c\) classes. A measure of sensitivity is then be defined as the difference between frequency distributions of input parameters per each class. Here, we propose to define the measure of sensitivity as the L1 norm difference between a class-conditional and marginal cdfs. Illustration of the procedure is shown in Figure 4.

A resampling procedure (hypothesis test) and a normalization of the L1 norm is performed to determine whether the parameter is influential on the response. Details are provided in Fenwick et al. (2014).

In reservoir modeling, one may find that no single parameter has a statistically significant influence on the response. One of the reasons is that the relationship between input parameters and responses is stochastic (due to spatial uncertainty) as mentioned above and that stochasticity confounds the relationship between input parameters and response. Another reason is that the relationship between parameters as a whole and the response is highly non-linear, hence certain combinations (two or more) of parameter values may influence the response. The above developed single-way sensitivity analysis has therefore been extended to quantify response sensitivity for parameter interactions. In a similar way, a measure of interaction sensitivity is the L1 norm difference between a conditional class-conditional and conditional cdfs (Figure 5, right).

![Figure 4. Distance-based GSA. Scoping runs are classified into 2 classes. Analysis of the parameter distribution of each class.](image)

![Figure 5. Example of class-conditional and marginal cdfs for parameter \(p_i\) (left). Example of conditional class-](image)
conditional and conditional cdfs for parameter $p_i$ and $p_j$ (right).

Results of the sensitivity values are presented in Figure 6 and 7.

![Figure 6. Sensitivity analysis for each parameter.](image)

![Figure 7. Sensitivity analysis for each parameter interaction.](image)

The analysis (Figure 6) shows that parameters $T_i$, watExp and krwMax are shown as influential. Interpreting interactions in complex models with multiple parameter types is not trivial, but there are a few things to note in Fig. XX. First, some of the interactions are asymmetric ($Kv/Kh|SOWCR$ is sensitive, whereas $SOWCR|Kv/Kh$ is not sensitive). Second, parameter $Kv/Kh$ was shown as not sensitive when considering the main effect only, whereas it is sensitive when combined with $SOWCR$.

Based on the measure of sensitivity of each parameter and parameter interactions, some of the parameters may be fixed to a constant value, because the prediction variable was shown to be insensitive to the parameter.

Note that one of the advantages of this technique is that, since the model responses are used only for classification, proxy models can be employed in this approach. In other words, what is important is that the responses correctly classify the models - the absolute accuracy of the proxy response itself is less consequential.

Now that the impacting factors are identified, the data is used to update the probability of each parameter and potentially reject some of the scenarios that were used to generate the initial set of scoping runs.

**UPDATING THE PROBABILITY OF SENSITIVE PARAMETERS**

Let’s consider that two parameters were defined as sensitive, for simplicity. One parameter is discrete (interpretation/scenario) and the other parameter is continuous. They are respectively denoted $D$ and $C$. Here we only provide a short summary of the proposed approach; details are provided in Scheidt et al., 2014.

A Bayesian formulation is used to update the probabilities of the uncertain parameters. In the presence of uncertain model parameters $D$ and $C$, the posterior distribution of models can be written as:

$$f(m|d_{obs}) = \int f(c|d_{obs})f_i(c|d_{obs})d_i \, dc$$

where $d_{obs}$ represent the new data.

The first part, $f(m|i_k,c)$, essentially calls for sampling using fixed values for parameters $I$ and $C$. The second part, $f_i(c|d_{obs})$ attempts to determine the joint uncertainty of the influential parameters given the data. Let us focus first on this second term, which can be decomposed as:

$$f_i(c|d_{obs}) = f_{i,D}(c|d_{obs})P(I=i_k|D=d_{obs})$$

Eq. 2 divides the task into determining first the updated probability of the discrete variable $P(I=i_k|D=d_{obs})$ given
the observed data and then, for each outcome $i_k$ of the
discrete variable, the densities of the continuous variable
given the observed data, $f_{C,Y,D}(c|i_k,d_{obs})$.

Often, in most practical cases with considerable
uncertainty, the term $P(i = i_k \mid D = d_{obs})$ in Eq. 2 is the most
critical: it requires investigating whether the geological
interpretation is in fact consistent with the data. Based on
their probability, some of the interpretations may be
removed from the study, hence rejected. For simplicity, we
only present a direct estimation method of the 2nd term in Eq.
2 using a combination of distance-based modeling and kernel
smoothing, thereby avoiding any complex and cpu-intensive
Markov chain Monte Carlo (McMC) methods. Details on how
to evaluate $f_{C,Y,D}(c|i_k,d_{obs})$ are provided in Scheidt et al.
2014.

**Modeling scenario probabilities and rejection**

The idea of scenario rejection based on data was
presented in Park et al. 2013. They employed the rejection
paradigm for reservoir modeling, instead of an accept/include
process (through history matching), as described above. In
the “rejection” process, without any history matching, the
probability of each scenario given the production data is
determined, and scenarios with low probability are rejected.
In the “sampling” step, they use an existing technique for
generating facies distribution constrained to production data
using a single given training image, namely the probability
perturbation method (although any other sampling technique
could be used). The proposed approach is fully Bayesian,
meaning, the scenarios (interpretations) are considered as
prior and the production data is used as a means to
determine a posterior by simple application of the Bayes’
rule. The approach is illustrated below.

Park et al. employed distance-based modeling to
calculate the probabilities of each scenario. No history
matching is done at this step; models are rejected based on
forward modeling only. From a set of prior models and their
associated responses, the distance between each model and
the observed response (water rate in their case) is computed.
The distance is defined as the difference between the
production curves. The models and the data are
subsequently projected in metric space. Figure 8 shows an
element of a metric space with 300 models of a simplified
version of the WCA test case. In this example, the type of
depositional scenario is the only uncertain parameter is
modeled using 3 different training images.

![Figure 8. MDS plot of water rate data for WCA and
type response.](image)

The probability of each scenario given the data is
modeled based on the density of points of each scenario
around the data using an adaptive kernel density estimation.
The procedure is illustrated in Figure 9.

![Figure 9. Updating probabilities of each TI using a
smoothing procedure.](image)

In Figure 9, we can see that the probability of TI1 is close
to 0, whereas TI2 and TI3 have much higher probabilities.
Based on this result, we can reject TI1 and retain only TI2 and
TI3 for history matching.

**History-matching by sampling**

We now proceed with sampling models for TI2 and TI3
constrained to production data using regional PPM. Using the
probability defined above, 29 posterior (history matched)
models were generated, 12 from TI2 and 17 from TI3
(consistent with updated probabilities). In regional PPM, the
field is decomposed into regions, either statically based on geological consideration or dynamically based on the streamline geometry which delineates drainage regions of the producing wells (Figure 10). The advantage is that perturbation is varied per region, resulting in faster convergence. The nature of the regional PPM algorithm is such that, although regions are used for model perturbation, they do not affect the reproduction of channel geometries or create artifacts at the region boundaries. Figure 11 (top) shows the quality of the match to production data. The total computational effort is 508 (PPM) + 180 (scoping) = 688 runs or \( \sim 24 \) flow simulations per model.

![Streamline geometry at final time step](image1)

**Figure 10.** (Left) Streamline simulation and (right) example of region geometry for one iteration in regional PPM

Although requiring considerable CPU, for the sake of this case study and for validating the proposed methodology, rejection sampling was applied. A large number of samples, namely 7,236 flow simulations, are needed to obtain the same amount of models as PPM. The outcome of this rejection sampler is twofold: (1) a set of 29 history-matched models and (2) updated TI probabilities. For those, we obtain:

\[
P(TI = ti1 | D = dobs) = 0.03 \text{ (1 model)}
\]

\[
P(TI = ti2 | D = dobs) = 0.34 \text{ (10 models)}
\]

\[
P(TI = ti3 | D = dobs) = 0.63 \text{ (18 models)}
\]

The updated TI probability obtained by rejection sampling is very similar to the one obtained by kernel smoothing in metric space. In addition, the quality of match is similar for both approached (Figure 11).

Methodology

![Rejection sampler](image2)

**Figure 11.** Comparing history match quality for methodology and rejection sampler.

*Forecasting a newly planned well.*

In order to compare the prediction uncertainty of the proposed methodology, production is forecast for an additional year, based on the rejection sampler and proposed methodology. Water rate predictions in the newly placed production well for both methodologies are shown in Figure 12. The P10, P50, and P90 quantiles show good agreement with the rejection sampler.

![P10-P50-P90 quantiles of water at the prediction year for rejection sampling and PPM](image3)

**Figure 12.** P10-P50-P90 quantiles of water at the prediction year for rejection sampling and PPM
CONCLUSION

An important component of the proposed approach is that it does not immediately focus on matching data, as there is a risk that the uncertainty represented by such ensemble is understated by ignoring important uncertainty in interpretations. Rather, the approach creates a framework which allows models to be built based on the purpose of the models, with a wide prior that incorporates uncertainty throughout the entire modeling process. An essential part of our workflow is the use of distance-based modeling, which allows us to directly incorporate the modeling purpose directly in the metric.

One of the advantages of the methodology is that it does not require to run a full physics simulator on all models. Sensitivity analysis can be performed using any proxy that provides an accurate classification of the response.

REFERENCES


